Selection of materials by multi-criteria methods applied to the side of a self-supporting structure for light vehicles

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Abstract: The selection of materials is an important stage in the design and development of products, but considering the enormous amount of materials available on the market that have different properties and characteristics, defining suitable and ideal alternatives is a difficult task. Within the automotive area there is a tendency to develop vehicles with greater efficiency and capacity, keeping aside the economic implications without underestimating the functionality of the materials. The use of multi-criteria methods (MCDM) allows the establishment of a reliable selection methodology, due to the interaction between each of the criteria with statistical methods that converge in a single solution. The methodology used in this study was based on the application of MCDM methods, and the comparison between them to determine a convergence in the alternatives of greater potential for the structural section of vehicles. Four methods were evaluated: TOPSIS, COPRAS, VIKOR, PROMETHEE II, obtaining that for all the methods the best material corresponds to the Martensitic Steel YS1200, being this the most appropriate one when fulfilling structural requirements, as well as providing a reduction of weight and price.

Keywords: lateral structure; weighting method; multi-criteria method; MCDM.

Reference to this paper should be made as follows: Loor, R.B.S., Martínez-Gómez, J., Rocha-Hoyos, J.C. and Cedeño, E.A.L. (2020) 'Selection of materials by multi-criteria methods applied to the side of a self-supporting structure for light vehicles', *Int. J. Mathematics in Operational Research*, Vol. 16, No. 2, pp.139–158.

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1 Introduction

A self-supporting structure is the most commonly used chassis configuration for passenger vehicles today. Its design is based on the concept of creating a surrounding metal structure made up of the union of elements of different shapes and thicknesses, forming a resistant box that supports itself and the mechanical elements fixed to it (Águeda and Casado, 2016; Kastillo et al., 2017; Heidarzade et al., 2016).

Ecuador is implementing a change in the country's production matrix, which is based mainly on the substitution of imports and the production of products with current quality standards, taking advantage of the agreements signed by Ecuador for their respective marketing in the country and in countries of the region in the short-term (Porras Blas, 2017).

The self-supporting structure is designed to absorb the forces due to driving (acceleration, deceleration, cornering, aerodynamic variables), in addition to absorbing the energy from deformation in the event of an accident. In response to a collision, the chassis is deformed creating 'deformation zones' designed to fold inwards, thus providing extended deceleration time and energy absorption, ensuring occupant life (Kershaw, 2017).

Among the items most commonly used by automobile manufacturers for the chassis are steel and aluminium, due to their strength and weight properties respectively. However, the primary use of one material over another can lead to an imbalance, preventing this objective from being achieved. Therefore, different alloys of materials or combinations of materials are currently used in order to obtain efficient, economic and adaptable design alternatives for the vehicle.

Therefore, the general objective of the work is to determine the appropriate material for the lateral section of the self-supporting structure using methods and tools that guarantee an objective and precise assessment in the decision making process.

2 Methods

2.1 Definition of the criteria

In the first instance, the requirements for the design and application of the material must be defined. It is considered that one of the primary objectives is the reduction of the weight of the lateral structural assembly, as well as its price. Within technical parameters are considered the elastic limit, ultimate strength, and percentage of elongation of the material (Acharya and Biswal, 2016; Azimi and Solimanpur, 2016; Villacís et al., 2015).

These criteria are ordered or optimised according to the type of influence they have on the case study, analysing each of these individually must be: since the weight is directly related to the density of the material, which is a characteristic of it and the greater it is in comparison to other heavier material will be, it is necessary that this value is minimal (Beltran et al., 2017; Godoy-Vaca et al., 2017; Chaloob et al., 2016).

The price of the material depends on many factors such as availability, volume of purchase, market volatility, etc. but in general there are stock markets that estimate the cost of each material, the most comfortable price should be sought without neglecting the technical requirements (Kastillo et al., 2015).

Table 1 Importance of defined criteria

Density (kg)	*Price ($/kg)*	*Young module (GPa)*	*Yield strength (MPa)*	*Ultimate tensile strength (MPa)*	*Total elongation (%)*
Minimum ↓	Minimum ↓	Maximum ↑	Maximum ↑	Maximum ↑	Minimum ↓

The elastic limit must be maintained at high margins, sufficient to withstand the impact load without reaching a deformation that affects the integrity of the occupants. The ultimate strength should be as high as possible, preventing material failure, while the elongation percentage should be a relatively low value, given that a permissible deformation is to be achieved but not exceeding the occupant space, Table 1 shows the objectives of maximisation and minimisation of criteria:

2.2 Optimisation models

For the process of optimising the selection of materials, a set of values subject to systems of equations called material indices are used. These indices relate different variables independently through equations in order to achieve a specific objective, which may be to seek the maximum or minimum value of a set of alternatives.

For the proposed case, the reduction of mass is considered (Ashby, 2011), as an important factor for the optimisation of the structural segment, so a mass ratio is established, as indicated in equation (1).

$$m_1 = m_2 = \rho V = AL_o\rho \tag{1}$$

where $m_1 y m_2$ represent the mass of the material as a function of geometric requirements, A is the area of the structural element cross-section, L_o is the length of the element and ρ es la densidad del material. The strength of a material is defined by equation (2).

$$\sigma_y = \frac{F}{A} \tag{2}$$

where σ_y represents the elastic limit of the material, F is a certain applied force. By clearing and replacing the area value of equation (2) with (1), an equation (3) is obtained that optimises the mass as a function of the resistance.

$$m_1 = FL_o\left(\frac{\rho}{\sigma_y}\right) \tag{3}$$

where FL_o corresponds to the functional requirements of the material and $\frac{\rho}{\sigma_y}$ is the material index to optimise the maximum resistance behaviour. Again, minimising the mass, by considering a beam-type linear element, it is considered to have a stiffness constant that is defined by equation (4).

$$S = \frac{EA}{L} \tag{4}$$

where S represents the stiffness constant of the element and E is the Young's module. Replacing the area of equation (4) with (1) gives an equation (5) that optimises the mass as a function of the yield strength.

$$m_2 = SL^2\frac{\rho}{E} \tag{5}$$

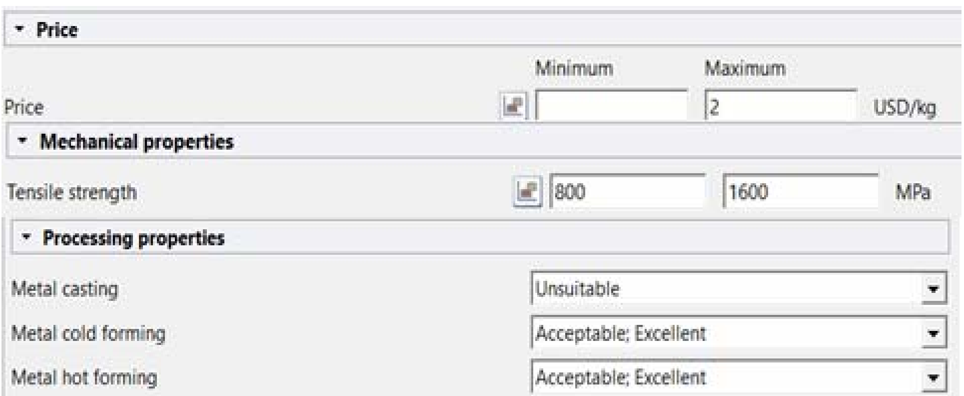
where SL^2 corresponds to the functional requirements of the material and ρ/E is the material index to optimise the maximum deformation behaviour.

2.3 Preselection of materials

For the preliminary selection of a series of materials suitable for the design of a self-supporting structure, the use of CES Edupack software was used, which has an enormous library of materials, in addition to including tools for comparison and revision of potential materials to be considered (Edupack, 2005).

Within the software tools is the limit option, which allows for limit values to be established to reduce the amount of materials to be considered to a finite number of considerable options, as shown in Figure 1. It is considered that within the cost value the material must be below 2 USD/kg, it must have a high resistance value so that values between 800–1,600 MPa were assigned, and it must have a good adaptability in the forming process.

Figure 1 Selected limitations for the selection of materials (see online version for colours)



Then, a graph is generated of the alternatives according to the minimisation of mass, the formulas described above are entered into a valuation interface that is responsible for calculating the masses based on the material indexes, a series of alternatives are presented represented by bubbles, in total 100 materials were obtained out of a total of 3,078.

However, this amount is still excessive to make an assessment, so the Pareto method is used to select the values closest to the abscissa of the masses, these are the smallest mass values compared to other alternatives, the brown line that crosses through the bubbles establishes the most appropriate alternatives for the weight reduction of the elements, this is observed in Figure 2.

These alternatives are possible potential solutions for this case study, in Tables 2 and 3 each of the materials and their determined properties are observed, in addition an identification code has been placed on each of them, to simplify and facilitate the use of them within the multi-criteria methods (MCDM).

2.4 Weighting methods and entropy method

Applying criteria weighting techniques, values are assigned to different criteria to indicate the relative importance in a decision making method, and these values are subsequently used during the processing and development of methods (Zardari, 2015; Villacreses et al., 2017).

Figure 2 Application of Pareto in material selection (see online version for colours)

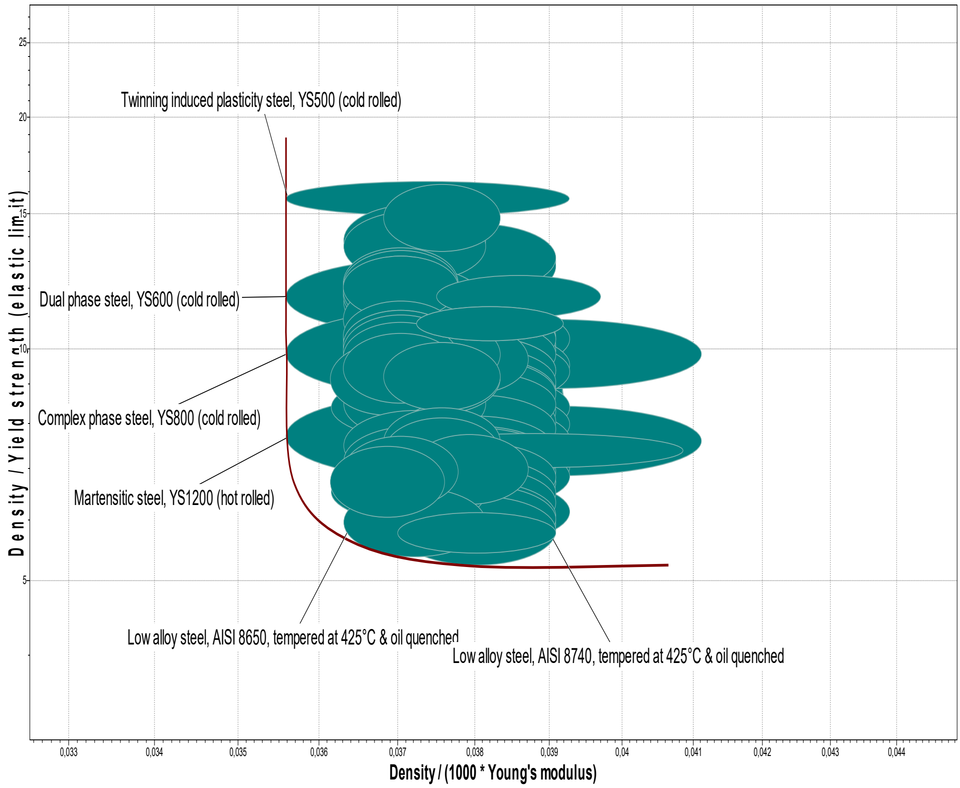


Table 2 Criteria determined with their respective values for each material

Price ($/kg)	*E Young module (GPa)*	*Yield strength (MPa)*	*Ultimate tensile strength (MPa)*	*Total elongation (%)*
1.325	210.5	500.5	981.5	55.0
0.645	210.5	675.0	1,055.0	13.5
1.265	210.5	800.0	1,090.0	10.0
0.780	210.5	1,025.0	1,300.0	5.5
0.735	211.0	1,325.0	1,450.0	12.0
0.745	206.5	1.360.0	1,435.0	13.0

Table 3 Candidate materials obtained at CES Edupack

Material	*Code*
Twinning induced plasticity steel, TS500 (cold rolled)	F1
Dual phase steel, YS600 (cold rolled)	F2
Complex phase steel, YS800 (cold rolled)	F3
Martensitic steel, YS1200 (hot rolled)	F4
Low alloy steel, AISI 8650, tempered at 425ºC and oil quenched	F5
Low alloy steel, AISI 8740, tempered at 425ºC and oil quenched	F6

The entropy method was used, which is defined as a measure of uncertainty in the information formulated using probability theories, having a wide distribution of data would represent more uncertainty than would represent peak or maximum values (Yilmaz and Harmancioglu, 2010). The steps for the application of the entropy method are as follows (Vatansever and Akgűl, 2018).

Step 1 Construction of the decision matrix.

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$

where x_{mn} corresponds to the values of the decision matrix.

Step 2 Normalisation of decision matrix, using equation (6).

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \tag{6}$$

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mn} \end{bmatrix}$$

where p_{ij} corresponds to the values of the standardised matrix m is the number of criteria evaluated.

Step 3 Calculation of entropy value, equations (7) and (8) are used.

$$k = \frac{1}{\ln m} \tag{7}$$

$$e_j = -k \sum_{i=1}^{m} p_{ij} \ln p_{ij} \tag{8}$$

where k is a constant that guarantees that $0 \leq e_j \leq 1$ ye_j is the value of entropy.

Step 4 Calculation of the degree of divergence, equation (9) is used.

$$d_j = 1 - e_j \tag{9}$$

where d_j is the degree of divergence.

Step 5 Obtaining the weights for criteria, using equation (10).

$$w_j = \frac{d_j}{\sum_{j=1}^{m} d_j} \tag{10}$$

where w_j is the weight of each criterion.

2.5 MCDMs and TOPSIS method

There are different types of MCDMs, each one has different qualities as far as the procedure and application are concerned, however, the point of convergence towards a reliable numerical solution is maintained in most of them, there are different variations or combinations of methods, four methods were used: TOPSIS, COPRAS, VIKOR and PROMETHEE II. Each of these is detailed below:

It is a technique that allows to evaluate the performance of alternatives, using as a concept the maximisation of the distance of negative ideal solutions and the minimisation of the distance of positive ideal solutions, so that it is possible to find acceptable solutions through the discretisation of variables (Al-Oqla, 2017; Martínez-Gómez and Narvaez, 2018). The steps used for the development of the TOPSIS method are detailed below.

Step 1 Normalised decision matrix, equations (11) and (12) are used, which correspond to the maximisation and minimisation of criteria respectively.

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a^2{}_{ij}}} \tag{11}$$

$$r_{ij} = 1 - \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a^2{}_{ij}}} \tag{12}$$

$$R_{ij} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}$$

where a_{ij} corresponds to the values of the decision matrix, r_{ij} corresponds to the values of the normalised matrix.

Step 2 Standardised weight matrix construction, using equation (13).

$$v_{ij} = w_n r_{mn}$$

$$V_{ij} = \begin{bmatrix} w_1 r_{11} & w_2 r_{12} & \cdots & w_n r_{1n} \\ w_1 r_{21} & w_2 r_{22} & \cdots & w_n r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_1 r_{m1} & w_2 r_{m2} & \cdots & w_n r_{mn} \end{bmatrix} \tag{13}$$

where w_n is the weight of each criterion, V_{ij} is the standardised weight matrix y v_{ij} is the standardised weight matrix and v_{ij} is the normalised value of each individual element.

Step 3 Determination of ideal positive and negative solutions, equations (14) and (15) are used.

$$A^* = \left\{ \max_i v_{ij} \middle| j \in J), \left(\min_i v_{ij} \middle| j \in J'\right) \right\} = \left\{ v_1{}^*, v_2{}^*, \cdots, v_n{}^* \right\} \tag{14}$$

$$A^- = \left\{ \max_i v_{ij} \,\middle|\, j \in J), \left(\min_i v_{ij} \,\middle|\, j \in J' \right) \right\} = \left\{ v_1^-, v_2^-, \cdots, v_n^- \right\} \quad (15)$$

where A^* y A^- correspond to the ideal positive and negative values respectively.

Step 4 Calculation of differences between measurements, equations (16) and (17) are used.

$$S_i^* = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^* \right)^2} \quad (16)$$

$$S_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^- \right)^2} \quad (17)$$

where S_i^* y S_i^- are the positive and negative separations respectively.

Step 5 Calculation of the proximity to the ideal solution, equation (18) is used.

$$C_i^* = \frac{S_i^-}{S_i^- + S_i^*} \quad (18)$$

where C_i^* is the relative proximity coefficient that represents a set of solutions, these are ordered from lowest to highest.

2.6 COPRAS method

This method considers and evaluates the performance of the alternatives against the different criteria and also the corresponding weightings of criteria. This method selects the best decision taking into account both the ideal solution and the anti-ideal solution (Chatterjee, 2013; Martínez-Gómez and Narvaez, 2018). The steps used for the development of the COPRAS method are detailed below.

Step 1 Normalised decision matrix, equation (19) is used.

$$r_{ij} = \frac{a_{ij}}{\sum_{i=1}^{m} a_{ij}} \quad (19)$$

where a_{ij} correspond to the values of the decision matrix y r_{ij} correspond to the values of the standard matrix.

Step 2 Standardised weight matrix construction. Similar to that applied in the TOPSIS method, using equation (12).

$$V_{ij} = \begin{bmatrix} w_1 r_{11} & w_2 r_{12} & \cdots & w_n r_{1n} \\ w_1 r_{21} & w_2 r_{22} & \cdots & w_n r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_1 r_{m1} & w_2 r_{m2} & \cdots & w_n r_{mn} \end{bmatrix}$$

Step 3 Determination of normalised weight values for beneficial and non-beneficial criteria using equations (20) and (21).

$$S_{+i} = \sum_{j=1}^{m} y_{+ij} \tag{20}$$

where y_{+ij} corresponds to a value for a beneficial criterion, S_{+i} corresponds to the sum of the values of y_{+i}.

$$S_{-i} = \sum_{j=1}^{m} y_{-ij} \tag{21}$$

where y_{-ij} corresponds to a value for a non-beneficial criterion and S_{-i} corresponds to the sum of the values of y_{-i}.

Step 4 Obtaining the relative priority of the alternatives, equation (22) is used.

$$Q_i = S_{+i} + \frac{\sum_{j=1}^{m} S_{-i}}{S_{-i} \sum_{j=1}^{m} \frac{1}{S_{-i}}} \tag{22}$$

where Q_i corresponds to a value for a non-beneficial criterion, the higher the value of Q_i, the higher the priority of the alternative.

Step 5 Determine the level of performance, equation (23) is used.

$$U_i = \frac{Q_i}{Q_{\max}} * 100 \tag{23}$$

where U_i represents a set of acceptable solutions, these are ranked from highest to lowest.

2.7 PROMETHEE II method

It is based on the preference function that can be used effectively for a finite set of sorting and selection alternatives on the basis of some mutually independent and contradictory criteria, using the comparison by pairs of alternatives the deviations shown by the alternatives according to each criterion are considered. The following are the steps used to develop the PROMETHEE II method.

Step 1 Normalised decision matrix, equations (24) and (25) are used, which correspond to the maximisation and minimisation of criteria respectively.

$$r_{ij} = \frac{a_{ij} - \min(a_{ij})}{\max(a_{ij}) - \min(a_{ij})} \tag{24}$$

$$r_{ij} = \frac{\max(a_{ij}) - a_{ij}}{\max(a_{ij}) - \min(a_{ij})} \tag{25}$$

where a_{ij} corresponds to the values of the decision matrix as well as their respective maximum and minimum values and r_{ij} corresponds to the values of the standard matrix.

Step 2 Calculation of preference functions, equations (26) and (27) are used.

If $R_{ij} \leq R_{i'j}$ then

$$p_j\left(i, i'\right) = 0 \tag{26}$$

If $R_{ij} > R_{i'j}$ then

$$p_j\left(i, i'\right) = R_{ij} - R_{i'j} \tag{27}$$

where $p_j(i, i')$ represents the preferred functions for locating deviations from smaller values and detecting approximations to acceptable solutions.

Step 3 Calculation of aggregated functions of preference, equation (28) is used.

$$\prod\left(i, i'\right) = \left[\frac{\sum_{j=1}^{m} W_j * P_j\left(i, i'\right)}{\sum_{j=1}^{m} W_j}\right] \tag{28}$$

Step 4 Determination of input and output flows, equations (29) and (30) are used.

$$\phi^{+}(i) = \frac{1}{n-1}\sum_{i'=1}^{n}\prod\left(i, i'\right) \tag{29}$$

$$\phi^{-}(i) = \frac{1}{n-1}\sum_{i'=1}^{n}\prod\left(i, i'\right) \tag{30}$$

where $\phi^{+}(i)$ y $\phi^{-}(i)$ are the input and output flows respectively, they express how dominant an alternative is over another.

Step 5 Determination of net flow, equation (31) is used.

$$\phi(i) = \phi^{+}(i) - \phi^{-}(i) \tag{31}$$

where $\phi(i)$ is the net flow, it expresses the best alternatives. They are ordered from the highest to the lowest.

2.8 VIKOR

The fundamental principle of this method is to focus on the classification and selection of a number of alternatives in the presence of conflicting criteria, which can be done by comparing the measure of proximity with the ideal alternatives (Yalçin and Ünlü, 2017). The following are the steps used for the development of the VIKOR method.

Step 1 Normalised decision matrix, equations (11) and (12) are used similarly to the TOPSIS method.

Step 2 Standardised weight matrix construction. Equation (13), similar to the TOPSIS method, is used.

Step 3 Calculation of indicators of ideal positive and negative solutions, these are similar to the TOPSIS method, but the form and consideration varies. Equations (32) and (33) are used.

$$A^* = \left\{\max_i f_{ij} \left| j \in J\right), \left(\min_i f_{ij} \left| j \in J'\right)\right\} = \left\{f_1^{\max}, f_2^{\max}, \cdots, f_n^{\max}\right\} \tag{32}$$

$$A^- = \left\{\max_i f_{ij} \left| j \in J\right), \left(\min_i f_{ij} \left| j \in J'\right)\right\} = \left\{f_1^{\min}, f_2^{\min}, \cdots, f_n^{\min}\right\} \tag{33}$$

where A^* y A^- are indicators of positive and negative solutions respectively, these express the importance of the value of one criterion over another.

Step 4 Calculation of measurement indicators. Equations (34) and (35) are used.

$$U_i = \sum_{j=1}^{n} \frac{w_f\left(f_j^{\max} - f_{ij}\right)}{\left(f_j^{\max} - f_j^{\min}\right)} \tag{34}$$

$$R_i = \max_j \left[\frac{w_f\left(f_j^{\max} - f_{ij}\right)}{\left(f_j^{\max} - f_j^{\min}\right)}\right] \tag{35}$$

where U_i y R_i are measurement indicators.

Step 5 Calculate optimal solutions. Equation (36) is used.

$$V_i = \frac{\alpha\left(U_i - U_{\min}\right)}{\left(U_{\max} - U_{\min}\right)} + \frac{(1-\alpha)\left(R_i - \$_{\min}\right)}{\left(R_{\max} - R_{\min}\right)} \tag{36}$$

where α represents a correlation constant, a value of 0.5 is generally used, and the maximum and minimum values of equations (34) and (35) are also determined, V_i represents the set of solutions obtained, these are ordered from highest to lowest.

3 Results

For the development of the weighting method, the entropy method was used, in which criteria such as density that influences the weight of the material, price, elastic modulus and elongation of the material were considered, so that it is relatively deformable, yield stress and ultimate stress that are high to withstand the impact conditions.

Table 4 Decision matrix (entropy)

Code	*Price ($/kg)*	*E Young module (GPa)*	*Yield strength (MPa)*	*Ultimate tensile strength (MPa)*	*Total elongation (%)*
F1	1.325	210.5	500.5	981.5	55.0
F2	0.645	210.5	675.0	1,055.0	13.5
F3	1.265	210.5	800.0	1,090.0	10.0
F4	0.780	210.5	1,025.0	1,300.0	5.5
F5	0.735	211.0	1,325.0	1,450.0	12.0
F6	0.745	206.5	1,360.0	1,435.0	13.0

As shown in Table 2, the density values are the same for all alternatives, so all materials are considered to meet the lightness requirement and are discarded from multi-criteria analysis. Tables 4, 5 and 6 show the standardised, normalised decision matrix and determining factors for determining the most relevant criteria.

Table 5 Normalised matrix (entropy)

Code	*Price ($/kg)*	*E Young module (GPa)*	*Yield strength (MPa)*	*Ultimate tensile strength (MPa)*	*Total elongation (%)*
F1	0.2411	0.1671	0.0880	0.1342	0.5046
F2	0.1174	0.1671	0.1187	0.1443	0.1239
F3	0.2302	0.1671	0.1407	0.1491	0.0917
F4	0.1419	0.1671	0.1803	0.1778	0.0505
F5	0.1338	0.1675	0.2330	0.1983	0.1101
F6	0.1356	0.1640	0.2392	0.1963	0.1193

Table 6 Calculation of values Ej, Dj y Wj (entropy)

Factor	*Price ($/kg)*	*E Young module (GPa)*	*Yield strength (MPa)*	*Ultimate tensile strength (MPa)*	*Total elongation (%)*
Ej	0.9765	1.0000	0.9674	0.9935	0.8205
Dj	0.0235	0.0000	0.0326	0.0065	0.1795
Wj	0.0970	0.0001	0.1347	0.0267	0.7415

For the value w_j corresponding to each of the weights for each criterion, it was determined that the most important attribute is that of elongation, because it is the greatest of all considered, followed subsequently by the stress to creep, price, ultimate stress and elastic modulus. An interesting aspect to observe is that, when comparing the initial values of the criteria, especially those values that have a relatively high distance between each of the alternatives, they are given a much higher level of importance, this is due to the dynamism and fluctuation that each criterion has, it would not make much sense to evaluate a condition almost identical to another, when there is greater imprecision and uncertainty in the remaining options. This would explain the tendency in the results obtained to converge towards the alternatives mentioned in the previous section.

Table 7 Normalised matrix (TOPSIS y VIKOR)

Code	*Price ($/kg)*	*E Young module (GPa)*	*Yield strength (MPa)*	*Ultimate tensile strength (MPa)*	*Total elongation (%)*
F1	0.4338	0.4094	0.2043	0.3251	0.0897
F2	0.7244	0.4094	0.2755	0.3494	0.7766
F3	0.4594	0.4094	0.3265	0.3610	0.8345
F4	0.6667	0.4094	0.4184	0.4306	0.9090
F5	0.6859	0.4103	0.5408	0.4802	0.8014
F6	0.6816	0.4016	0.5551	0.4753	0.7848

For the development of the MCDMs, each of the steps mentioned within the methodology were used, for the TOPSIS method, equations (11) and (12) were used to determine the standardised matrix as shown in Table 7, and equation (13) to determine the standardised weight matrix as shown in Table 8, equations (14) and (15) to determine the ideal and non-ideal solutions as shown in Table 9, equations (16) and (17) to establish the separation rates as shown in Table 10 and equation (18) to determine the proximity coefficients of each alternative, obtaining that the best material corresponds to Martensitic steel in Table 11.

Table 8 Standardised matrix of weights obtained (TOPSIS y VIKOR)

Code	*Price ($/kg)*	*E Young module (GPa)*	*Yield strength (MPa)*	*Ultimate tensile strength (MPa)*	*Total elongation (%)*
F1	0.0421	0.00002547	0.0275	0.0087	0.0665
F2	0.0703	0.00002547	0.0371	0.0093	0.5758
F3	0.0446	0.00002547	0.0440	0.0097	0.6187
F4	0.0647	0.00002547	0.0564	0.0115	0.6740
F5	0.0665	0.00002553	0.0729	0.0128	0.5942
F6	0.0661	0.00002499	0.0748	0.0127	0.5819

Table 9 Ideal positive and negative solution (TOPSIS y VIKOR)

A^*	0.0421	0.00002553	0.0748	0.0128	0.6740
A^-	0.0703	0.00002499	0.0275	0.0087	0.0665

Table 10 Distances between ideal positive and negative solutions (TOPSIS)

Code	S_i^*	S_i^-
F1	0.6093	0.0282
F2	0.1089	0.5094
F3	0.0634	0.5531
F4	0.0292	0.6082
F5	0.0835	0.5297
F6	0.0951	0.5176

Table 11 Relative proximity to ideal solution and ranking (TOPSIS)

Code	C_i^*	*Ranking*
F1	0.0442	6
F2	0.8238	5
F3	0.8972	2
F4	0.9542	1
F5	0.8639	3
F6	0.8448	4

In the COPRAS method, equation (19) was used to determine the normalised matrix, as described in Table 12; equation (12) was used to obtain the standardised weight matrix, as shown in Table 13; equations (20) and (21) are used to determine the normalised values of benefit and cost, as shown in Table 14, equation (22) is used to obtain the priority alternatives, can be seen in Table 15, and equation (23) is used to obtain the performance level of each alternative, as shown in Table 16, finding that Martensitic steel is the best option.

Table 12 Normalised matrix (COPRAS)

Code	*Price ($/kg)*	*E Young module (GPa)*	*Yield strength (MPa)*	*Ultimate tensile strength (MPa)*	*Total elongation (%)*
F1	0.2411	0.1671	0.0880	0.1342	0.5046
F2	0.1174	0.1671	0.1187	0.1443	0.1239
F3	0.2302	0.1671	0.1407	0.1491	0.0917
F4	0.1419	0.1671	0.1803	0.1778	0.0505
F5	0.1338	0.1675	0.2330	0.1983	0.1101
F6	0.1356	0.1640	0.2392	0.1963	0.1193

Table 13 Standardised matrix of weights obtained (COPRAS)

Code	*Price ($/kg)*	*E Young module (GPa)*	*Yield strength (MPa)*	*Ultimate tensile strength (MPa)*	*Total elongation (%)*
F1	0.0234	0.00001040	0.0119	0.0036	0.3741
F2	0.0114	0.00001040	0.0160	0.0039	0.0918
F3	0.0223	0.00001040	0.0190	0.0040	0.0680
F4	0.0138	0.00001040	0.0243	0.0048	0.0374
F5	0.0130	0.00001042	0.0314	0.0053	0.0816
F6	0.0131	0.00001020	0.0322	0.0052	0.0884

Table 14 Summation of weights obtained from beneficial and non-beneficial criteria (COPRAS)

S_{+1}	S_{+2}	S_{+3}	S_{+4}	S_{+5}	S_{+6}
0.0155	0.0199	0.0230	0.0291	0.0367	0.0375
S_{-1}	S_{-2}	S_{-3}	S_{-4}	S_{-5}	S_{-6}
0.3975	0.1032	0.0904	0.512	0.0946	0.1016

Table 15 Relative priorities for each alternative (COPRAS)

Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
0.0488	0.1483	0.1697	0.2882	0.1769	0.1680

Table 16 Relative proximity to ideal solution and ranking (COPRAS)

Code	*Performance level*	*Ranking*
F1	16.94	6
F2	51.48	5
F3	58.90	3
F4	100.00	1
F5	61.39	2
F6	58.31	4

In the PROMETHEE II method, equations (24) and (25) were used to determine the standardised matrix, as shown in Table 17; equations (26) and (27) were used to obtain the preference functions, as shown in Table 18; equation (28) is used to determine the values of the aggregated functions, as shown in Table 19, equations (29), (30) and (31) are used to obtain the input, output and net value values of each alternative, as shown in Table 20, finding that Martensitic steel is the best alternative for this method.

Table 17 Normalised matrix (PROMETHEE II)

Code	*Price ($/kg)*	*E Young module (GPa)*	*Yield strength (MPa)*	*Ultimate tensile strength (MPa)*	*Total elongation (%)*
F1	0.0000	0.8889	0.0000	0.0000	0.0000
F2	1.0000	0.8889	0.2030	0.1569	0.8384
F3	0.0882	0.8889	0.3485	0.2316	0.9091
F4	0.8015	0.8889	0.6102	0.6798	1.0000
F5	0.8676	1.0000	0.9593	1.0000	0.8687
F6	0.8529	0.0000	1.0000	0.9680	0.8485

Table 18 Matrix of preference or pair functions (PROMETHEE II)

Alternative	*Price ($/kg)*	*E Young module (GPa)*	*Yield strength (MPa)*	*Ultimate tensile strength (MPa)*	*Total elongation (%)*
$p_1(1, 2)$	0.0000	0.0000	0.0000	0.0000	0.0000
$p_1(1, 3)$	0.0000	0.0000	0.0000	0.0000	0.0000
$p_1(1, 4)$	0.0000	0.0000	0.0000	0.0000	0.0000
$p_1(1, 5)$	0.0000	0.0000	0.0000	0.0000	0.0000
$p_1(1, 6)$	0.0000	0.8889	0.0000	0.0000	0.0000
$p_2(2, 1)$	1.0000	0.0000	0.2030	0.1569	0.8384
$p_2(2, 3)$	0.9118	0.0000	0.0000	0.0000	0.0000
$p_2(2, 4)$	0.1985	0.0000	0.0000	0.0000	0.0000
$p_2(2, 5)$	0.1324	0.0000	0.0000	0.0000	0.0000
$p_2(2, 6)$	0.1471	0.8889	0.0000	0.0000	0.0000
$p_3(3, 1)$	0.0882	0.0000	0.3485	0.2316	0.9091
$p_3(3, 2)$	0.0000	0.0000	0.1454	0.0747	0.0707
$p_3(3, 4)$	0.0000	0.0000	0.0000	0.0000	0.0000
$p_3(3, 5)$	0.0000	0.0000	0.0000	0.0000	0.0404
$p_3(3, 6)$	0.0000	0.8889	0.0000	0.0000	0.0606
$p_4(4, 1)$	0.8015	0.0000	0.6102	0.6798	1.0000
$p_4(4, 2)$	0.0000	0.0000	0.4072	0.5229	0.1616
$p_4(4, 3)$	0.7132	0.0000	0.2618	0.4482	0.0909
$p_4(4, 5)$	0.0000	0.0000	0.0000	0.0000	0.1313
$p_4(4, 6)$	0.0000	0.8889	0.0000	0.0000	0.1515
$p_5(5, 1)$	0.8676	0.1111	0.9593	1.0000	0.8687
$p_5(5, 2)$	0.0000	0.1111	0.7563	0.8431	0.0303

Table 18 Matrix of preference or pair functions (PROMETHEE II) (continued)

Alternative	*Price ($/kg)*	*E Young module (GPa)*	*Yield strength (MPa)*	*Ultimate tensile strength (MPa)*	*Total elongation (%)*
$p_5(5, 3)$	0.7794	0.1111	0.6108	0.7684	0.0000
$p_5(5, 4)$	0.0662	0.1111	0.3490	0.3202	0.0000
$p_5(5, 6)$	0.0147	1.0000	0.0000	0.0320	0.0202
$p_6(6, 1)$	0.8529	0.0000	1.0000	0.9680	0.8485
$p_6(6, 2)$	0.0000	0.0000	0.7970	0.8111	0.0101
$p_6(6, 3)$	0.7647	0.0000	0.6515	0.7364	0.0000
$p_6(6, 4)$	0.0515	0.0000	0.3898	0.2882	0.0000
$p_6(6, 5)$	0.0000	0.0000	0.0407	0.0000	0.0000

Table 19 Matrix of added functions or pairs (PROMETHEE II)

Code	*Price ($/kg)*	*E Young module (GPa)*	*Yield strength (MPa)*	*Ultimate tensile strength (MPa)*	*Total elongation (%)*
F1	0.0000	0.0000	0.0000	0.0000	0.0001
F2	0.0000	0.0884	0.0193	0.0128	0.0143
F3	0.0740	0.0000	0.0000	0.0300	0.0450
F4	0.1887	0.1838	0.0000	0.0974	0.1124
F5	0.1469	0.1785	0.0620	0.0000	0.0173
F6	0.1366	0.1817	0.0652	0.0055	0

Table 20 Determination of input, output, and net worth flows (PROMETHEE II)

Alternatives	*φ+*	*φ–*	*φ-φ*	*Ranking*
F1	0.00001	0.8325	–0.8324	6
F2	0.1770	0.1092	0.0678	4
F3	0.1769	0.1265	0.0505	5
F4	0.3004	0.0293	0.2711	1
F5	0.2578	0.0291	0.2287	2
F6	0.2523	0.0378	0.2145	3

While in the VIKOR method, the same methodology of TOPSIS was used until step 3 of this method, so the values obtained for the standardised and weighted matrix, as well as the ideal and anti-ideal values are those indicated in Tables 7, 8 and 9, then equations (34), (35) and (36) are applied to determine the measurement indicators and solution conditions, as shown in Table 21, obtaining that the best material to be selected corresponds to Martensitic steel.

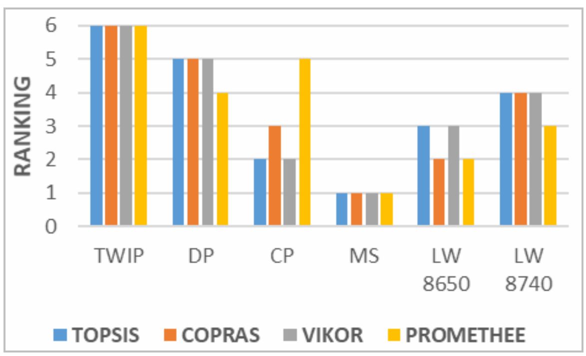
All materials that are placed first for the MCDMs described above converge to a single solution, which provides a greater degree of reliability in considering the Martensitic steel material as the most optimal for the design of the structural side segment of light vehicles.

Multi-criteria analysis methods are a very useful tool for determining reliable and safe selection alternatives, regardless of the methodology used, a common solution can be found between the different methods applied. The entropy method was found to be the best criterion with 74% elongation, 13% yield stress, 10% price, 2% ultimate stress, and less than 1% modulus of elasticity. Applying this method of entropy in the multi-criteria analysis it is obtained that the alternative F4 or (MS) corresponding to Martensitic steel is the best option, for all the methods used (TOPSIS, COPRAS, VIKOR and PROMETHEE II), as indicated in Figure 3.

Table 21 Determination of measurement indicators (VIKOR)

Ui	*Ri*	*Vi*
0.9030	0.7415	1.0000
0.3468	0.1198	0.1678
0.1843	0.0878	0.0373
0.1388	0.0777	0.0000
0.1870	0.0974	0.0463
0.1960	0.1123	0.0635

Figure 3 Comparison of MCDMs (see online version for colours)



Among other materials are that 8,650 low alloy steels are considered as the second choice for COPRAS and PROMETHEE II methods and seconds for complex phase steels in COPRAS and VIKOR methods, the other materials are placed irregularly and below the second place so they are discarded as a possible alternative. In this section you should discuss and explain each of your results. It should contrast and relate them to the content of previously published articles and theories that may or may not support their results.

4 Conclusions

This article presents the application of multi-criteria analysis methods, which were used to determine the appropriate material for the structural segment of light vehicles. The comparison of solution alternatives is presented, showing that they are an effective tool in the selection of materials, regardless of the type of application.

By applying multi-criteria analysis methods, it was possible to determine a suitable material for application in the lateral structural elements of light vehicles. Which has strength properties at an acceptable level, an affordable price in the market, is light, has not excessive deformation levels, all these aspects can define the material as an ideal alternative for this type of application. The weighting methods are a complement to define the level of importance of a series of attributes according to the distance and correlation that exists between each of their values. The multi-criteria analysis methods allow to face decision problems where there are criteria that are difficult to quantify by means of the traditional analysis of human evaluation.

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